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# Absolute stability of a class of nonlinear systems with nonpositive linear parts

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The positivity and absolute stability of a class of nonlinear continuous-time and discrete-time systems with nonpositive linear part are addressed. Necessary and sufficient conditions for the positivity of this class of nonlinear systems are established. Sufficient conditions for the absolute stability of this class of nonlinear systems are also given.

**Key words:** absolute stability, positive, nonlinear, discrete-time, continuous-time, systems

## 1. Introduction

A dynamical system is called positive if its trajectory starting from any nonnegative initial state remains forever in the positive orthant for all nonnegative inputs. An overview of state of the art in positive theory is given in the monographs and papers [1, 2, 6, 11, 12]. Variety of models having positive behavior can be found in engineering, economics, social sciences, biology and medicine.

The stability of linear and nonlinear standard and positive fractional systems has been addressed in [3–6, 8, 15, 16, 20–23]. The stabilization of positive descriptor fractional systems has been investigated in [10, 11, 20, 21]. The superstable linear systems have been addressed in [17, 18]. Positive linear systems with different fractional orders have been introduced in [14, 13] and their stability has been analyzed in [3, 20].

The absolute stability of a class of positive nonlinear systems has been investigated in [7]. In this paper the positivity and absolute stability of a class of nonlinear continuous-time and discrete-time systems with nonpositive linear parts will be addressed.

The paper is organized as follows. In section 2 some preliminaries concerning positivity and stability of linear systems are recalled. The positivity and absolute stability of positive continuous-time nonlinear systems with nonpositive linear

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parts is investigated in section 3 and of positive discrete-time nonlinear systems in section 4. Concluding remarks are given in section 5.

The following notation will be used:  $\mathfrak{R}$  – the set of real numbers,  $\mathfrak{R}^{n \times m}$  – the set of  $n \times m$  real matrices,  $\mathfrak{R}_+^{n \times m}$  – the set of  $n \times m$  real matrices with nonnegative entries and  $\mathfrak{R}_+^n = \mathfrak{R}_+^{n \times 1}$ ,  $M_n$  – the set of  $n \times n$  Metzler matrices (real matrices with nonnegative off-diagonal entries),  $I_n$  – the  $n \times n$  identity matrix,  $A^T$  – the transpose of matrix  $A$ .

## 2. Preliminaries

Consider the continuous-time linear system

$$\dot{x} = Ax + Bu, \quad (1a)$$

$$y = Cx, \quad (1b)$$

where  $x = x(t) \in \mathfrak{R}^n$ ,  $u = u(t) \in \mathfrak{R}^m$ ,  $y = y(t) \in \mathfrak{R}^p$  are the state, input and output vectors and  $A \in \mathfrak{R}^{n \times n}$ ,  $B \in \mathfrak{R}^{n \times m}$ ,  $C \in \mathfrak{R}^{p \times n}$ .

**Definition 1** [6, 12] *The continuous-time linear system (1) is called (internally) positive if  $x(t) \in \mathfrak{R}_+^n$ ,  $y(t) \in \mathfrak{R}_+^p$ ,  $t \geq 0$  for any initial conditions  $x(0) \in \mathfrak{R}_+^n$  and all inputs  $u(t) \in \mathfrak{R}_+^m$ ,  $t \geq 0$ .*

**Theorem 1** [6, 12] *The continuous-time linear system (1) is positive if and only if*

$$A \in M_n, \quad B \in \mathfrak{R}_+^{n \times m}, \quad C \in \mathfrak{R}_+^{p \times n}. \quad (2)$$

**Definition 2** [6, 12] *The positive continuous-time system (1) for  $u(t) = 0$  is called asymptotically stable if*

$$\lim_{t \rightarrow \infty} x(t) = 0 \quad \text{for any } x(0) \in \mathfrak{R}_+^n. \quad (3)$$

**Theorem 2** [6, 12] *The positive continuous-time linear system (1) for  $u(t) = 0$  is asymptotically stable if and only if one of the following equivalent conditions is satisfied:*

1. *All coefficient of the characteristic polynomial*

$$p_n(s) = \det[I_n s - A] = s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0 \quad (4)$$

*are positive, i.e.  $a_i > 0$  for  $i = 0, 1, \dots, n-1$ .*

2. *There exists strictly positive vector  $\lambda^T = [\lambda_1 \dots \lambda_n]^T$ ,  $\lambda_k > 0$ ,  $k = 1, \dots, n$  such that*

$$A\lambda < 0 \quad \text{or} \quad \lambda^T A < 0. \quad (5)$$

Consider the discrete-time linear system

$$x_{i+1} = Ax_i + Bu_i, \quad i \in Z_+ = \{0, 1, \dots\}, \quad (6a)$$

$$y_i = Cx_i, \quad (6b)$$

where  $x_i \in \mathfrak{R}^n$ ,  $u_i \in \mathfrak{R}^m$ ,  $y_i \in \mathfrak{R}^p$  are the state and input vectors and  $A \in \mathfrak{R}^{n \times n}$ ,  $B \in \mathfrak{R}^{n \times m}$ ,  $C \in \mathfrak{R}^{p \times n}$ .

**Definition 3** [6, 12] The discrete-time linear system (6) is called (internally) positive if  $x_i \in \mathfrak{R}_+^n$ ,  $y_i \in \mathfrak{R}_+^p$ ,  $i \in Z_+$  for any initial conditions  $x_0 \in \mathfrak{R}_+^n$  and all inputs  $u_i \in \mathfrak{R}_+^m$ ,  $i \in Z_+$ .

**Theorem 3** [6, 12] The discrete-time linear system (6) is positive if and only if

$$A \in \mathfrak{R}_+^{n \times n}, \quad B \in \mathfrak{R}_+^{n \times m}, \quad C \in \mathfrak{R}_+^{p \times n}. \quad (7)$$

**Definition 4** [6, 12] The positive discrete-time system (6) for  $u_i = 0$  is called asymptotically stable if

$$\lim_{i \rightarrow \infty} x_i = 0 \quad \text{for any } x_0 \in \mathfrak{R}_+^n. \quad (8)$$

**Theorem 4** [6, 12] The positive discrete-time linear system (6) for  $u_i = 0$  is asymptotically stable if and only if one of the following equivalent conditions is satisfied:

1. All coefficient of the characteristic polynomial

$$p_n(z) = \det[I_n(z+1) - A] = z^n + a_{n-1}z^{n-1} + \dots + a_1z + a_0 \quad (9)$$

are positive, i.e.  $a_i > 0$  for  $i = 0, 1, \dots, n-1$ .

2. There exists strictly positive vector  $\lambda^T = [\lambda_1 \ \dots \ \lambda_n]^T$ ,  $\lambda_k > 0$ ,  $k = 1, \dots, n$  such that

$$(A - I_n)\lambda < 0 \quad \text{or} \quad \lambda^T(A^T - I_n) < 0. \quad (10)$$

If the matrix  $(A - I_n)$  is nonsingular then we can choose  $\lambda = (A - I_n)^{-1}c$ , where  $c \in \mathfrak{R}^n$  is strictly positive.

### 3. Absolute stability of positive continuous-time nonlinear systems with nonpositive linear parts

Consider the nonlinear continuous-time system shown in Fig. 1 and described by the equations

$$\dot{x} = Ax + Bu, \quad u = f(e), \quad (11a)$$

$$y = Cx, \tag{11b}$$

where  $x = x(t) \in \mathfrak{R}^n$ ,  $u = u(t) \in \mathfrak{R}^m$ ,  $y = y(t) \in \mathfrak{R}^p$  are the state, input and output vectors of the system  $A \in \mathfrak{R}^{n \times n}$ ,  $B \in \mathfrak{R}^{n \times 1}$ ,  $C \in \mathfrak{R}^{1 \times n}$  and the characteristic  $f(e)$  of the nonlinear element (Fig. 2) satisfies the condition

$$0 < f(e) < ke, \quad 0 < k < \infty. \tag{12}$$

It is assumed that  $A$  is not a Metzler matrix.

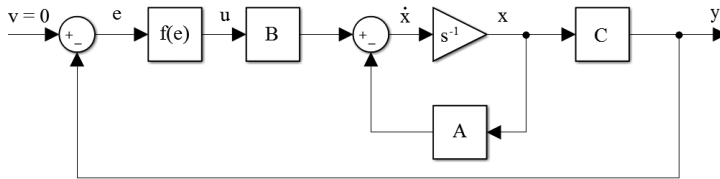


Figure 1: Nonlinear system

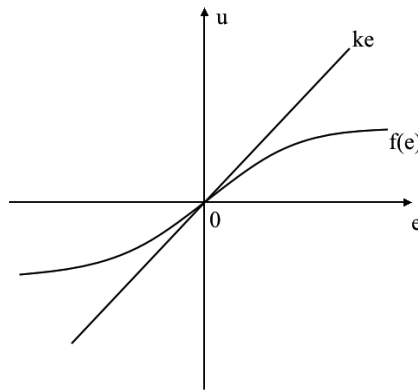


Figure 2: Characteristic of nonlinear element

**Definition 5** The nonlinear system (11) is called (internally) positive if  $x(t) \in \mathfrak{R}_+^n$ ,  $y(t) \in \mathfrak{R}_+^p$ ,  $t \geq 0$  for any initial conditions  $x(0) \in \mathfrak{R}_+^n$  and all inputs  $u(t) \in \mathfrak{R}_+$ ,  $t \geq 0$ .

**Theorem 5** The nonlinear system (11) is positive if and only if

$$A \in M_n, \quad B \in \mathfrak{R}_+^{n \times 1}, \quad C \in \mathfrak{R}_+^{1 \times n} \tag{13a}$$

and

$$f(e) \geq 0 \text{ for } e \geq 0 \quad \text{and} \quad f(-e) < 0 \text{ for } -e < 0. \tag{13b}$$

**Proof.** It is well-known [11] that if  $u = f(e) \in \mathfrak{X}_+, t \geq 0$  then  $x(t) \in \mathfrak{X}_+, t \geq 0$  for  $x(0) \in \mathfrak{X}_+^n$  if and only if  $A \in M_n$  and  $B \in \mathfrak{X}_+^{n \times 1}$ . From (11b) for  $t = 0$  we have  $y(0) = Cx(0) \in \mathfrak{X}_+$  for  $x(0) \in \mathfrak{X}_+^n$  if and only if  $C \in \mathfrak{X}_+^{1 \times n}$ .  $\square$

**Definition 6** The positive nonlinear system (11) is called absolutely stable if  $x(t) \in \mathfrak{X}_+, t \geq 0$  and

$$\lim_{t \rightarrow \infty} x(t) = 0 \quad \text{for any } x(0) \in \mathfrak{X}_+^n. \quad (14)$$

The Metzler matrix  $A \in M_n$  is called Hurwitz Metzler matrix if its all eigenvalues  $\lambda_k$  satisfy the condition  $\operatorname{Re} \lambda_k < 0, k = 1, \dots, n$ .

**Theorem 6** The positive nonlinear system (11) is absolutely stable if:

1.  $A \in M_n$  is the Hurwitz Metzler matrix,

$$B \in \mathfrak{X}_+^{n \times 1}, \quad C \in \mathfrak{X}_+^{1 \times n}. \quad (15)$$

2. The nonlinear characteristic  $f(e)$  satisfy the condition (12).

**Proof.** Proof is based on the Lyapunov method for positive systems. As a candidate of the Lyapunov function it is assumed the linear function of the state vector  $x(t) \in \mathfrak{X}_+, t \geq 0$

$$V(x(t)) = \lambda^T x(t), \quad \lambda^T = [\lambda_1 \ \dots \ \lambda_n], \quad \lambda_k > 0, \quad k = 1, \dots, n. \quad (16)$$

Using (16) and (11a) we obtain

$$\dot{V}(x) = \lambda^T \dot{x}(t) = \lambda^T [Ax(t) + Bf(e)] < 0 \quad (17)$$

since by (15) and (5)

$$\lambda^T A < 0 \quad \text{and} \quad f(-e) < 0 \quad \text{for } -e < 0 \quad \text{and} \quad t \geq 0. \quad (18)$$

Therefore, the positive nonlinear system (11) is absolutely stable if the conditions 1 and 2 of Theorem 6 are satisfied.  $\square$

To modify the properties of the linear part (of the matrix  $A$ ) the feedback with a gain  $k$  has been applied to the system (Fig. 3a).

Using the well-known results we transform the nonlinear system to the form shown in Fig. 3b with the new characteristic of nonlinear element

$$\bar{f}(e) = f(e) - ke \quad (19)$$

and the new state matrix of the linear part

$$\bar{A} = A + kBC. \quad (20)$$

The gain  $k$  is chosen so that the matrix  $\bar{A} \in M_n$  is asymptotically stable.

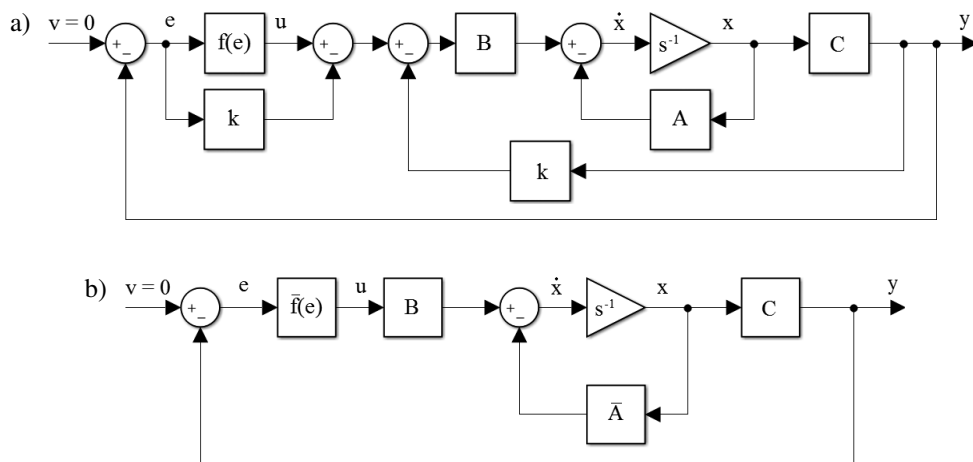


Figure 3

**Example 1.** Consider the nonlinear continuous-time system (11) with the matrices

$$A = \begin{bmatrix} -2 & -1 \\ 1 & -2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C_1 = [0 \ 20] \quad (21)$$

and the characteristic of nonlinear element satisfying the condition (12).

The matrix  $A$  is not a Metzler matrix since one of its off diagonal entry is negative. The gain coefficient  $k$  is chosen  $k = 0.1$ , so that the matrix

$$\bar{A} = A + kBC = \begin{bmatrix} -2 & -1 \\ 1 & -2 \end{bmatrix} + 0.1 \begin{bmatrix} 0 & 20 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \quad (22)$$

is Hurwitz Metzler matrix with the eigenvalues  $s_1 = -1, s_2 = -3$ .

The characteristic  $\bar{f}(e)$  of nonlinear element satisfies the condition

$$\bar{f}(e) = f(e) - ke = f(e) - 0.1e \geq 0 \quad \text{for } e \geq 0 \quad (23a)$$

and

$$\bar{f}(e) < 0 \quad \text{for } e < 0. \quad (23b)$$

Therefore, the nonlinear system with (21) and the characteristic of nonlinear element satisfying the condition (23) is absolutely stable.

**4. Absolute stability of positive discrete-time nonlinear systems with nonpositive linear parts**

Consider the nonlinear discrete-time system shown in Fig. 4 and described by the equations

$$x_{i+1} = Ax_i + Bu_i, \quad u_i = f(e_i), \quad i \in Z_+ = \{0, 1, \dots\}, \quad (24a)$$

$$y_i = Cx_i, \quad (24b)$$

where  $x_i \in \mathbb{R}^n$ ,  $u_i \in \mathbb{R}^m$ ,  $y_i \in \mathbb{R}^p$  are the state, input and output vectors of the system  $A \in \mathbb{R}^{n \times n}$ ,  $B \in \mathbb{R}^{n \times 1}$ ,  $C \in \mathbb{R}^{1 \times n}$  and the characteristic  $f(e_i)$  of the nonlinear element (Fig. 5) satisfies the condition

$$0 < f(e_i) < ke_i, \quad 0 < k < \infty. \quad (25)$$

It is assumed that  $A = [a_{ij}]$  is not a positive matrix, i.e.  $a_{ij} < 0$  for some  $i, j = 1, \dots, n$ .

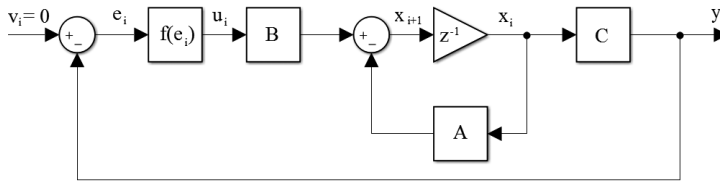


Figure 4: Nonlinear system

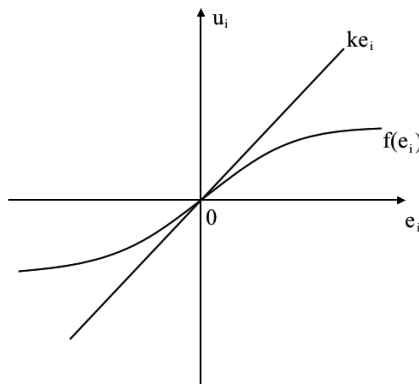


Figure 5: Characteristic of nonlinear element

**Definition 7** The nonlinear system (24) is called (internally) positive if  $x_i \in \mathbb{R}_+^n$ ,  $y_i \in \mathbb{R}_+^p$ ,  $i \in Z_+$  for any initial conditions  $x_0 \in \mathbb{R}_+^n$  and all inputs  $u_i \in \mathbb{R}_+$ ,  $i \in Z_+$ .

**Theorem 7** *The nonlinear system (24) is positive if and only if*

$$A \in \mathfrak{R}_+^{n \times n}, \quad B \in \mathfrak{R}_+^{n \times 1}, \quad C \in \mathfrak{R}_+^{1 \times n} \quad (26a)$$

and

$$f(e_i) \geq 0 \quad \text{for } e_i \geq 0 \quad \text{and} \quad f(-e_i) < 0 \quad \text{for } -e_i < 0, \quad i \in Z_+. \quad (26b)$$

**Proof.** It is well-known [11] that if  $u_i = f(e_i) \in \mathfrak{R}_+, i \geq 0$  then  $x_i \in \mathfrak{R}_+^n, i \in Z_+$  for  $x_i \in \mathfrak{R}_+^n$  if and only if  $A \in \mathfrak{R}_+^{n \times n}$  and  $B \in \mathfrak{R}_+^{n \times 1}$ . From (24b) for  $i = 0$  we have  $y_0 = Cx_0 \in \mathfrak{R}_+$  for  $x_0 \in \mathfrak{R}_+^n$  if and only if  $C \in \mathfrak{R}_+^{1 \times n}$ .  $\square$

**Definition 8** *The positive nonlinear system (24) is called absolutely stable if  $x_i \in \mathfrak{R}_+^n, i \in Z_+$  and*

$$\lim_{i \rightarrow \infty} x_i = 0 \quad \text{for any } x_0 \in \mathfrak{R}_+^n. \quad (27)$$

The matrix  $A \in \mathfrak{R}_+^{n \times n}$  is called Schur matrix if its all eigenvalues  $z_i$  satisfy the condition

$$|z_i| < 1, \quad i = 1, \dots, n. \quad (28)$$

**Theorem 8** *The positive nonlinear system (24) is absolutely stable if:*

1.  $A \in \mathfrak{R}_+^{n \times n}$  is the Schur matrix,

$$B \in \mathfrak{R}_+^{n \times 1}, \quad C \in \mathfrak{R}_+^{1 \times n}. \quad (29)$$

2. The nonlinear characteristic  $f(e_i)$  satisfy the condition (25).

**Proof.** Proof is based on the Lyapunov method for positive discrete-time systems. As a candidate of the Lyapunov function it is assumed the linear function of the state vector  $x_i \in \mathfrak{R}_+^n, i \in Z_+$

$$V(x_i) = \lambda^T x_i, \quad \lambda^T = [\lambda_1 \quad \dots \quad \lambda_n], \quad \lambda_k > 0, \quad k = 1, \dots, n. \quad (30)$$

Using (30) and (24a) we obtain

$$\Delta V(x_i) = V(x_{i+1}) - V(x_i) = \lambda^T (x_{i+1} - x_i) = \lambda^T (A - I_n)x_i + Bf(e_i) < 0 \quad (31)$$

since by (29) and (10)

$$\lambda^T (A - I_n) < 0 \quad \text{and} \quad f(-e_i) < 0 \quad \text{for } i \in Z_+. \quad (32)$$

Therefore, the positive nonlinear system (24) is absolutely stable if the conditions 1 and 2 of Theorem 8 are satisfied.  $\square$



To modify the properties of the linear part (of the matrix  $A$ ) the feedback with a gain  $k$  has been applied in a similar way as for the continuous-time case (Fig. 3a). As the result of this transformation we obtain the system shown in Fig. 6, with the new characteristic of nonlinear element

$$\bar{f}(e_i) = f(e_i) - ke_i, \quad i \in Z_+ \quad (33)$$

and the new state matrix of the linear part

$$\bar{A} = A + kBC. \quad (34)$$

The gain  $k$  is chosen so that the matrix  $A \in \mathfrak{R}_+^{n \times n}$  is asymptotically stable.

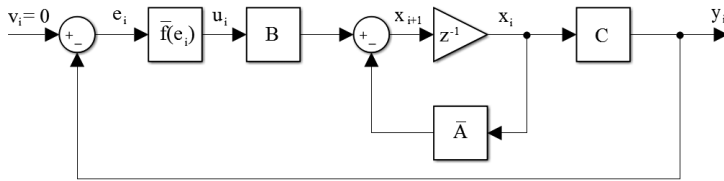


Figure 6

**Example 2.** Consider the nonlinear discrete-time system (24) with the matrices

$$A = \begin{bmatrix} 0.2 & 0.2 & -0.4 \\ 0 & 0.1 & -0.4 \\ -0.3 & 0 & -0.3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix} \quad (35)$$

and the characteristic of the nonlinear element satisfying the condition (25). The matrix  $A$  (given by (35)) has negative entries and the linear system with the matrices (35) is not positive one.

The gain coefficient  $k$  is chosen  $k = 0.2$ , so that the matrix

$$\begin{aligned} \bar{A} = A + kBC &= \begin{bmatrix} 0.2 & 0.2 & -0.4 \\ 0 & 0.1 & -0.4 \\ -0.3 & 0 & -0.3 \end{bmatrix} + 0.2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 0.4 & 0.2 & 0 \\ 0 & 0.3 & 0.2 \\ 0.1 & 0 & 0.5 \end{bmatrix} \end{aligned} \quad (36)$$

has nonnegative entries and is asymptotically stable since the characteristic polynomial

$$\det[I_3(s+1) - \bar{A}] = \begin{vmatrix} s+0.6 & -0.2 & 0 \\ 0 & s+0.7 & -0.2 \\ -0.1 & 0 & s+0.5 \end{vmatrix} = s^3 + 1.8s^2 + 1.07s + 0.206 \quad (37)$$

has positive coefficients (Theorem 4).

The characteristic  $\bar{f}(e_i)$  of nonlinear element satisfies the condition

$$\bar{f}(e_i) = f(e_i) - ke_i = f(e_i) - 0.2e_i \geq 0 \quad \text{for } e_i \geq 0 \quad (38a)$$

and

$$\bar{f}(e_i) < 0 \quad \text{for } e_i < 0, \quad i \in Z_+. \quad (38b)$$

Therefore, the nonlinear system with the matrix  $\bar{A}$  given by (36) and the characteristic of nonlinear element satisfying (38) is absolutely stable.

## 5. Concluding remarks

The positivity and absolute stability of a class of nonlinear continuous-time and discrete-time systems with nonpositive linear parts have been addressed. Necessary and sufficient conditions for the positivity of the nonlinear systems have been established (Theorems 5 and 7). Sufficient conditions for the absolute stability of the nonlinear systems have been also obtained (Theorems 6 and 8). The considerations have been illustrated by numerical examples. The presented results can be extended to multi-inputs multi-outputs nonlinear systems. The considerations can be also extended to fractional nonlinear systems with the same fractional order and with different fractional orders.

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