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Selection of Sound Insulating Elements in Hydraulic Excavators Based Onidentification of Vibroacoustic Energy Propagation Paths

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In spite of the fact that standardizing operations and increased awareness of hazards led to a significant improvement of vibroacoustic climate of operator's stands of new machines, their long-term operation – often under difficult conditions – leads to a fast degradation of acoustic qualities of machines. Temporary operations performed during surveys and periodical overhauls are rarely effective, due to the lack of any guidelines. In this situation the authors propose the algorithm for selection of eventual screens or sound absorbing and sound insulating partitions, utilizing the measuring procedure aimed at identification, at the operator's stand, of main noise components originated from various sources. On the basis of this procedure, the vibroacoustic energy propagation paths in the machine was estimated.

Keywords: noise and vibration, propagation path, coherence function.

1. Introduction

A lot of emphasis is currently put on the employees protection at work stands. It concerns, among others, operators of earth work machines. Since driving systems of this type of machines are characterised with high vibroactivity, minimising noise and vibration hazards belongs to the most important structural problems and modernisation programs of existing machines. The applied methods of suppression noises and vibrations can be divided into two groups: active and passive methods.

A principle of operation of active damping methods can be generally described as generation of signal components of time waveforms being phase-shifted by a half of a period in relation to noise and vibration components subjected to damping. In this case the waves decay effect as a result of interferences is utilised. Such technique can be only successful in case of time synchronisation which in case of random effects requires a system with a very short reaction time as well as maintaining the same (opposite) propagation directions. These difficulties are the cause for which active methods are used only in certain specific applications.

Passive techniques of noise and vibration damping can be divided, from the point of view of the physics of the effect, into the following groups:

- minimisation of noise and vibration sources;
- changing vibroacoustic energy into another form (e.g. thermal);
- changing the propagation direction of acoustic field and dissipating its energy outside the noise protected area.

The last two groups consist in designing structures to be put on the vibroacoustic energy propagation path. The methodology of machine noise and vibrations damping can be put in order according to the following scheme:

- localisation and description of basic sources;
- identification of the model of the vibroacoustic energy propagation;
- determination frequency characteristics of the designed damping systems;
- designing damping elements according to the determined characteristics.

In search for effective tools of the computer-aided designing of structures minimising vibroacoustic hazards caused by earth work machines, the present authors assumed the following:

1. The basis of computational algorithms of insulating structures should be the determination of main

propagation paths of the vibroacoustic energy between noise and vibration sources and the operator's work stand – on the grounds of the identified mathematical model.

2. The proposed mathematical model of the vibroacoustic energy propagation in a machine should be identifiable by a simple measurement procedure and should constitute the basis for the computer-aided selection of vibroinsulating elements.
3. The model identification should be performed by means of a procedure allowing for the separation components originated from individual sources from the signals recorded at the system output (noise and vibrations in the operator's cabin) – under the assumption that the investigated system can be non-linear.

The investigated problem is widely analysed and discussed in the world literature, especially by scientific research groups related to the automotive industry (e.g. Society of Automotive Engineers, SAE). Some commercial programs analysing signals in time and/or frequency domains were also developed. They are using the method of modal analysis or the Finite Element Method (FEM) for modelling and determination of parameters characterising propagation of vibroacoustic energy. Several methods described in the literature are based on the energy propagation analysis using measurements of the acoustic particle velocity, including:

- Source Path Contribution (SPC) consists in performing a phased summation of partial responses from all noise and vibration paths to give total tactile and acoustic responses under specific operating loads at a given frequency or RPM (Source Path Contribution software, 2010). A similar principle of operation has the Transfer Path Analysis (TPA).
- Statistical Energy Analysis (SEA) is used to predict the sound transmission loss, the radiation resistance and the vibration amplitude of a partition. Conformity between theory and experiment is shown to be good. The "mass-law" sound transmission is seen to be due to non-resonant modal vibration, while the increased transmission in the coincidence region is due to resonant modal vibration (Crocker, Price, 1969). To solve a noise and vibration problem with the SEA, the system is divided into a number of components (such as plates, shells, beams, and acoustic cavities) that are coupled together in various combinations. Each component can support a number of different propagating wave types (e.g. bending, longitudinal, and shear wave fields in a thin isotropic plate). From the SEA point of view, the reverberant field of each wave field represents an orthogonal store of energy and thus is represented as a separate energy degree of freedom in the SEA equations.

2. Theoretical considerations

Vibration-noise effects occurring in real systems, such as e.g. machines for earth works, can be presented as a continuous exchange of the mechanical vibrations energy into noise and vice versa. The energy originated from correlated and non-correlated sources of vibrations and noises passes through the complicated machine structure undergoing linear (and often non-linear) transformations. This energy arrives to various points, e.g. machine operator stand, as a sum in which fractions of individual sources do not have to be proportional (and usually are not) to their power.

The structural model describing these effects is presented in Fig. 1.

The most important issue in the realised task is determination of the influence of individual factors (localised and identified vibroacoustic energy sources) on noise and vibrations influencing the operator. This is a difficult task, since we analyse recorded signals of effects occurring in a complicated, non-linear, inner machine field with several independent sources of vibroacoustic energy. From the point of view of the need of decreasing vibrations and noise levels, it is necessary to estimate fractions of individual sources in the total energy at the machine point of interest or in its vicinity by determining propagation paths properties.

Because of the above-mentioned non-linear effects, disturbances, and hardly identifiable (invisible) changes of a vibroacoustic machine structure during

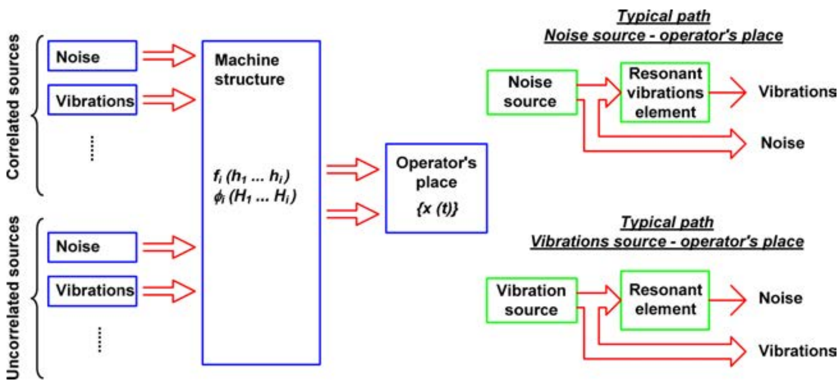

Fig. 1. Structural model describing vibroacoustic energy propagation in machines.

operation, detailed models are far from the reality and difficult to identify. According to the authors, much better effects can be achieved by building a simplified model (partially of a 'black box' type) well identified on the grounds of input and output values (Batko *et al.*, 2008). Therefore the authors have proposed their own method of investigating the vibroacoustic energy propagation in machines for earth works.

In order to determine the propagation paths properties, the acoustic pressure changes in the vicinity of identified sources of vibroacoustic energy should be recorded (in case of propagation in air) as well as the accelerations of vibrations at points of attaching these sources to the carrying system (in case of a propagation in the machine structure). Simultaneously, the acoustic pressure changes and accelerations of vibrations influencing the machine operator should be recorded.

The proposed method of analysis is based on the application of the autocorrelation and ordinary coherence functions. Correlation and coherence functions belong to the classic analytical methods used, among other things, in investigation of determined signals disturbed by random noises. In the time domain, the autocorrelation function allows to determine the time cohesion between neighbouring fragments of the analysed process, shifted by various time values. The ordinary coherence function is a spectral measure of the cohesion of processes in the frequency domain. However, these are not the only properties of these functions. The autocorrelation function is a superposition of the determined process and random noise being the result of disturbances from other processes or from the measurement. Due to that, it is possible to determine the fraction related to random disturbances and to the determined part in the total signal energy. The autocoherence value depends, among other things, on the random disturbance value and on non-linearity of the analysed system.

In order to limit the influence of harmonics of similar frequencies and originated from various vibroacoustic energy sources, application of an algorithm using the given standard signal was proposed. One of the essential advantages of the proposed method is improving the resolution analysis in the frequency domain in comparison with classic spectral methods.

The simplest description of the vibroacoustic energy propagation path can be supplemented by presenting the total effect of weakly non-linear disturbances in the form of one or a few unknown functions determined in the identification process and taken into account in the model, either in an additive or a multiplicative way. For one dominating source, it is reduced to the following description, in the frequency domain:

$$Y(f) = \Im\{y(t)\} = X(f)\cdot H(f) + \varPhi(f) + \varPsi(f) \tag{1}$$

or

$$Y(f) = \Im\{y(t)\} = (X(f)\cdot H(f)) * \varPhi(f) + \varPsi(f), \tag{2}$$

where $Y(f)$ – Fourier transform of the output signal $y(t)$, $X(f)$ – excitation process (Fourier transform of the input signal $x(t)$), $H(f)$ – transmittance of the propagation path, $\varPhi(f)$ – a function being the result of non-linear disturbance and determined in the identification process, $\varPsi(f)$ – influence of external disturbances (random effects).

After simple transformation, Eq. (1) can be brought to the product form (Dąbrowski, 1992; Crocker *et al.*, 2007):

$$Y(f) = X(f)\cdot H(f)\cdot \varPhi^*(f)\cdot \varPsi^*(f). \tag{3}$$

This form allows to subject both sides of Eq. (3) to the operator changing the component amplitudes of spectra into values expressed in decibels. Assuming that the total transmittance can be presented as the product of transmittances of individual elements of series systems (elements causing a vibroacoustic energy decrease), reduces the task to the equation of level decreases:

$$\begin{aligned} L_c(f) = {} & L_s(f) + \sum \Delta L_i(f) + \Delta\varPhi^*(f) \\ & + \Delta\varPsi^*(f), \end{aligned} \tag{4}$$

where $L_c(f)$ – signal level at the output, $L_s(f)$ – signal level originated from the dominating source of the vibroacoustic energy, $\Delta L_i(f)$ – signal level changes at individual elements of the series system, $\Delta\varPhi^*(f)$ – signal level change at the output, caused by non-linear disturbances and determined in the identification process, $\Delta\varPsi^*(f)$ – error of the description caused by various random disturbances.

The problem becomes complicated when there are several vibroacoustic energy sources. In this case, the main aim of modelling is obtaining the independent description of each propagation path. Mutual couplings, resulting, among other things, from the system non-linearity, usually make it impossible to uncouple equations and thus the problem becomes much more complex when the number of the described paths increases.

Let us consider the case of two weakly correlated vibroacoustic energy sources (Fig. 2). In such model we have to assume three unknown functions to be introduced into the model in one of the two ways,

$$\begin{aligned} Y(f) = {} & X_1(f)\cdot H_1(f) * \varPhi_1(f) \\ & + X_2(f)\cdot H_2(f) * \varPhi_2(f) + \varPhi_{12}(f) + \varPsi(f) \end{aligned} \tag{5}$$

or

$$\begin{aligned} Y(f) = {} & (X_1(f)\cdot H_1(f) + \varPhi_1(f) \\ & + X_2(f)\cdot H_2(f) + \varPhi_2(f)) * \varPhi_{12}(f) + \varPsi(f), \end{aligned} \tag{6}$$

where $\varPhi_1(f)$ – correction for the non-linear disturbance for the first source, $\varPhi_2(f)$ – correction for the non-linear disturbance for the second source, $\varPhi_{12}(f)$ – correction for the mutual dependence of the determined transmittances $H_1(f)$ and $H_2(f)$ resulting, among other things, from the system non-linearity.

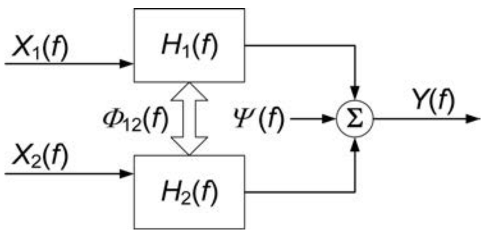

Fig. 2. Model of the system with two non-correlated sources.

Determination of unknown functions $\Phi_i(f)$, in both cases, is possible only by performing at least triple measurements at three points (for each of the active sources and for both simultaneously). When separate measurements of sources are impossible, measurements at various conditions of the machine work can be done (e.g. at different rotational speeds of the driving system), however the error caused by random component fraction will be larger. Thus, any simple generalisation of such models, especially for larger number of sources (partially correlated), is virtually impossible.

3. Example of the model of the vibroacoustic energy propagation path

The problem of separation vibroacoustic propagation paths becomes even more difficult in machines in which the driving system consists of a few structurally identical systems and similar frequency characteristics of the generated vibroacoustic energy.

Let us analyse the vibroacoustic energy propagation in a two-motor machine for which, during operation, noise measurements were performed in motor chambers and vibration measurements on motor supports. At the same time, the noise was recorded at the operator's stage (Fig. 3).

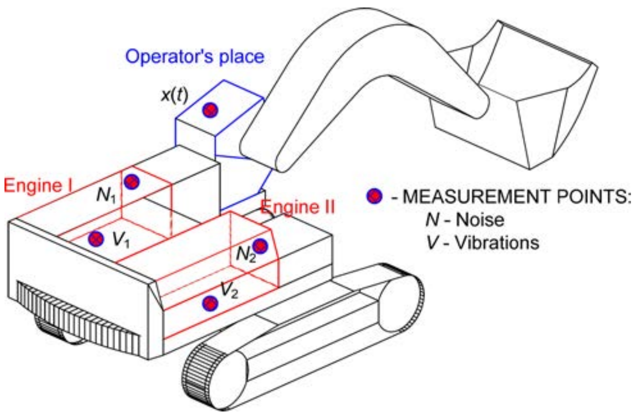

Fig. 3. Distribution of measuring points in a two-motor machine (using the example of a hydraulic excavator).

The model of the vibroacoustic energy propagation paths for this type of objects is presented in Fig. 4.

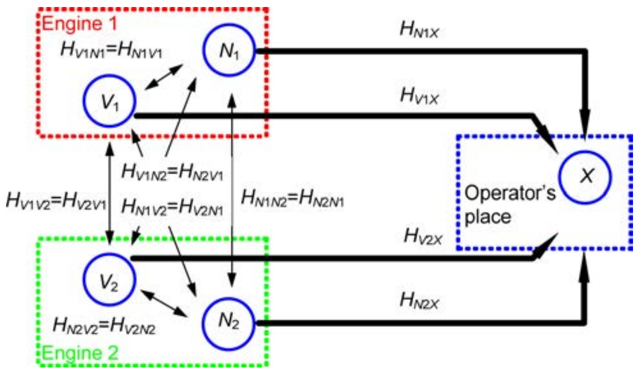

Fig. 4. Example of the propagation model of the vibroacoustic energy for a two-motor machine.

This model is described by the following equations system:

$$\begin{aligned}
N_1^* \cdot H_{N1X} &= (N_1 + N_2 \cdot H_{N2N1} + V_1 \cdot H_{V1N1} \\
&\qquad + V_2 \cdot H_{V2N1}) \cdot H_{N1X} = Y_{N1X}, \\
N_2^* \cdot H_{N2X} &= (N_2 + N_1 \cdot H_{N1N2} + V_2 \cdot H_{V2N2} \\
&\qquad + V_1 \cdot H_{V1N2}) \cdot H_{N2X} = Y_{N2X}, \\
V_1^* \cdot H_{V1X} &= (V_1 + V_2 \cdot H_{V2V1} + N_1 \cdot H_{N1V1} \\
&\qquad + N_2 \cdot H_{N2V1}) \cdot H_{V1X} = Y_{V1X}, \\
V_2^* \cdot H_{V2X} &= (V_2 + V_1 \cdot H_{V1V2} + N_2 \cdot H_{N2V2} \\
&\qquad + N_1 \cdot H_{N1V2}) \cdot H_{V2X} = Y_{V2X}, \\
\sum_i Y_{NiX} &+ \sum_j Y_{VjX} + \sum_i \Phi_{NiX} \\
&\qquad + \sum_j \Phi_{VjX} + \Psi = X,
\end{aligned} \tag{7}$$

where N_1^*, N_2^* – spectral density of the noise signals power recorded near sources, V_1^*, V_2^* – spectral density of the vibration signals power, recorded near sources, N_1, N_2 – spectral density of the noise signals power, V_1, V_2 – spectral density of the vibration signals power, X – spectral density of the noise signals power measured in the operator's cabin, H_{NiX} – transmittances of the noise propagation paths between individual sources, H_{NiX} – transmittances of the vibration propagation paths between individual sources, Y_{NiX}, Y_{NiX} – noise components at the operator's stand originated from various propagation paths, $\sum_i \Phi_{NiX}$, $\sum_j \Phi_{VjX}$ – total corrections for non-linear disturbances in the system, Ψ – influence of external disturbances (random effects).

When comparing, in the identification process, the model with the recorded noise and vibrations signals, we can try to determine the energy originated from individual sources. As can be noticed, the model described by Eqs. (7) has much more unknowns than equations which requires either theoretical determination of a part of them (which is usually impossible) or undertaking efforts to simplify the model or looking for additional relationships.

Before we set about discussing the latter possibility, let us try to simplify the model by assuming that the mutual influence of vibrations of one of the sources on noises of the second source is negligible. The model simplified in such a way is presented in Fig. 5.

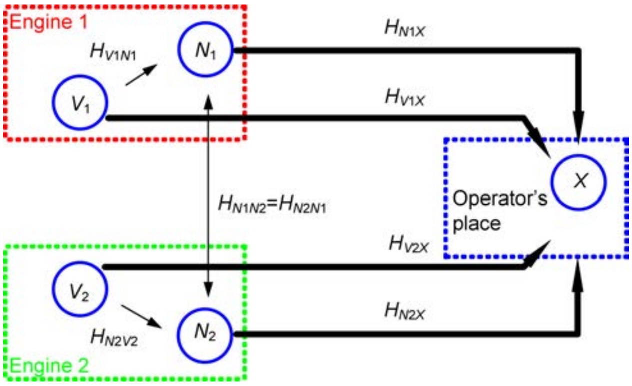

Fig. 5. Simplified model of the vibroacoustic energy propagation for a two-motor machine.

Individual propagation paths can be described as follows:

$$\begin{aligned}
N_1^* \cdot H_{N1X} &= (N_1 + N_2 \cdot H_{N2N1} \\
&\quad + V_1 \cdot H_{V1N1}) \cdot H_{N1X} = Y_{N1X}, \\
N_2^* \cdot H_{N2X} &= (N_2 + N_1 \cdot H_{N1N2} \\
&\quad + V_2 \cdot H_{V2N2}) \cdot H_{N2X} = Y_{N2X}, \\
V_1 \cdot H_{V1X} &= Y_{V1X}, \\
V_2 \cdot H_{V2X} &= Y_{V2X}, \\
\sum_i Y_{NiX} + \sum_j Y_{VjX} &+ \sum_i \Phi_{NiX} \\
&+ \sum_j \Phi_{VjX} + \Psi = X.
\end{aligned} \tag{8}$$

After rearranging Eq. (8) and taking into account that

$$\sum_{i,j} Y_{i,j} = \sum_{i,j} U_{i,j} \quad (Y_{i,j} \neq U_{i,j}),$$

equation of energy fractions from individual sources in the total energy of the signal measured at the operator's stand will be as follows:

$$\begin{aligned}
N_1 \cdot (H_{N1X} + H_{N1N2} \cdot H_{N2X}) &= U_{N1X} + \Phi_{N1X}, \\
N_2 \cdot (H_{N2X} + H_{N2N1} \cdot H_{N1X}) &= U_{N2X} + \Phi_{N2X}, \\
V_1 \cdot (H_{V1X} + H_{V1N1} \cdot H_{N1X}) &= U_{V1X} + \Phi_{V1X}, \\
V_2 \cdot (H_{V2X} + H_{V2N2} \cdot H_{N2X}) &= U_{V2X} + \Phi_{V2X}, \\
\sum_i U_{NiX} + \sum_j U_{VjX} + \sum_i \Phi_{NiX} & \\
+ \sum_j \Phi_{VjX} + \Psi &= X,
\end{aligned} \tag{9}$$

where U_{NiX} – energy fraction of the noise source in the noise measured in the operator's cabin, U_{VjX} – energy fraction of the vibration source in the noise measured in the operator's cabin, and other symbols have the same meaning as in Eq. (7).

In spite of introduced simplifications there are still too many unknowns in Eq. (9). Thus, the only possibility of obtaining a reliable model is looking for new relationships.

As the solution to this problem, the authors assumed the precise identification of high-energy harmonic components of the investigated vibroacoustic signals. In order to do that, application of the normal coherence function was proposed for the determination of energy fractions originated from individual sources for the selected harmonic components. Decomposing functions $\sum U_{NiX}$ and $\sum U_{VjX}$ into individual components allows to reduce each equation from the system (9) to the form (4).

4. Application of the normal coherence function

As it follows from previous considerations, the problem of separation of components representing vibroacoustic processes originated from various sources is the crucial problem solution of which enables the mathematical model identification.

Values of the normal coherence function $\gamma_{xy}^2(f)$ can be interpreted as a part of the output signal energy derived from harmonic components of the input signal for successive frequencies f. This function is often applied in tasks involving identification of energy propagation paths by means of the spectral method (together with the multiple coherence function) (Bendat, Piersol, 1993; 2010). Each of the energy propagation paths, in the system with n independent sources, can be treated as the single-input independent series system. The function of normal coherence for such system ca be written as

$$\gamma_{xy}^2(f) = \frac{|G_{xy}(f)|^2}{G_{xx}(f) \cdot G_{yy}(f)}, \tag{10}$$

where $G_{xy}(f)$ – cross-spectral function density of the input $x(t)$ and output $y(t)$ signal, $G_{xx}(f)$ – autospectral density function of the input signal $x(t)$, $G_{yy}(f)$ – autospectral density function of the output signal $y(t)$. Since $|G_{xy}(f)|^2 \leq G_{xx}(f) \cdot G_{yy}(f)$, the normal coherence function can take values from the range $\langle 0, 1\rangle$.

However, such application of the normal coherence function does not allow to solve the task. The description of the paths of the vibroacoustic energy propagation, even in the simplified form, contains too many unknowns. Moreover, the coherence function value depends on several other factors, including non-linear disturbances, random effects, and frequency components coming from very close frequency values (differences are comparable with resolution Δf of the spectral analysis).

If, in the course of the model identification, we focus on the analysis of harmonic components of the highest energies, we will be able to use properties of the coherence function in a different way. The principle of the proposed computational algorithm is based on the determination the coherence function between individual investigated signals and the harmonic signal which was generated with the assumed parameters (among other things, the frequency value). Treating the signal generated this way as the reference one, we can look for harmonic components in the investigated signals originated from energy sources and recorded at the machine operator's stand (Fig. 6). Frequency values of the sought harmonic components can be found by analysing operation of individual elements of the driving system.

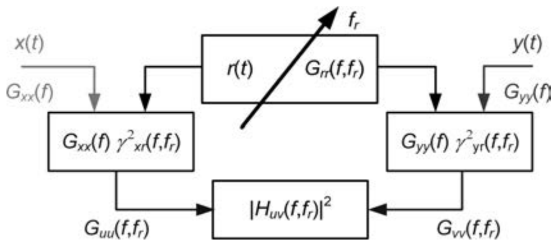

Fig. 6. Schematic presentation of the algorithm for determination of the coherence function with the intermediate signal (reference).

Let us analyse the operation of the proposed algorithm (Dziurdz, 2000; 2013). The measured signals are represented by spectral densities $G_{xx}(f)$ and $G_{yy}(f)$. By calculating coherence functions between the recorded signals and the reference signal of a spectral density $G_{rr}(f)$,

$$\begin{aligned}\gamma^2_{xr}(f) &= \frac{|G_{xr}(f)|^2}{G_{xx}(f)\cdot G_{rr}(f)},\\ \gamma^2_{yr}(f) &= \frac{|G_{yr}(f)|^2}{G_{yy}(f)\cdot G_{rr}(f)},\end{aligned} \tag{11}$$

it is possible to determine the energy being transferred by harmonic components of real signals (accurately at least for the part related to the linear response of the system):

$$\begin{aligned}G_{uu}(f) &= G_{xx}(f)\cdot\gamma^2_{xr}(f),\\ G_{vv}(f) &= G_{yy}(f)\cdot\gamma^2_{yr}(f).\end{aligned} \tag{12}$$

Dividing the obtained values by each other we obtain the coefficient of amplification squared:

$$\frac{G_{vv}(f)}{G_{uu}(f)} = |H_{uv}(f)|^2\,. \tag{13}$$

Moreover, this algorithm allows, by the controlled change of the reference signal frequency, for a more accurate determination of frequencies of harmonic components of the investigated signals and separation of harmonic components of similar frequencies.

The verification of the operation effectiveness of the proposed method was carried out for signals recorded in a real object (hydraulic excavator) in which the driving motor was working at a constant rotational speed of approx. 2000 rpm. This corresponds to the rotational frequency of approx. 33.33 Hz. Accelerations of vibrations recorded at the motor support and on the operator's cabin floor were the analysed signals.

At first, let us determine the actual rotational frequency of the motor (with an accuracy possible to obtain when the algorithm is applied). Determination of more accurate rotational frequency is essential insofar as the knowledge of kinematic relations allows for better determination of frequencies of vibroacoustic signals components originated from the driving system.

Narrow-band spectra of accelerations of vibrations at selected points of the machine are presented in Fig. 7. Based on them, the preliminarily determined rotational frequency of the motor was 33.57 Hz (approx. 2014 rpm).

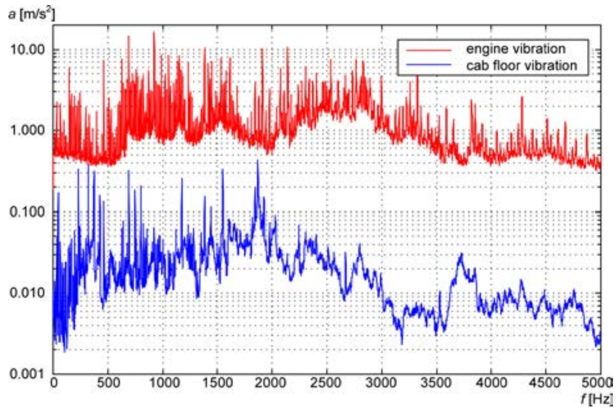

Fig. 7. Spectra of acceleration of vibrations at selected points of the machine.

The results of more accurate analysis of the rotational frequency signal component are presented in Fig. 8. Overall results of the performed calculations are presented in Table 1.

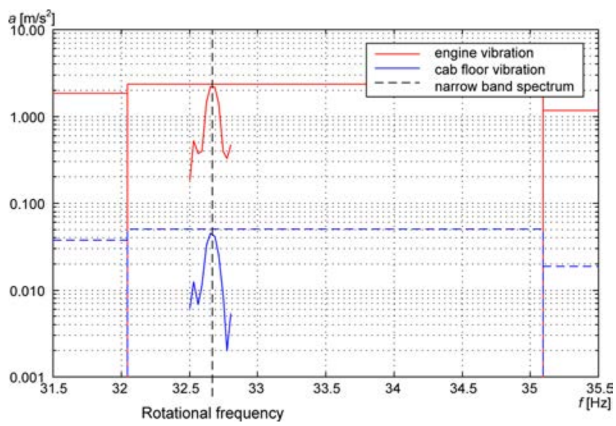

Fig. 8. Example of the vibration accelerations analysis for the signal component of the rotational frequency.

Table 1. Analytical results for the signal component of the rotational frequency.

	Spectral line	Determined harmonic component
Signal frequency [Hz]	33.57	32.65
Accelerations of vibrations of the motor support [m/s^2]	2.339	2.211
Accelerations of vibrations of the operator's cabin floor [m/s^2]	0.051	0.045
Coefficient of amplification [-]	$21.8{\cdot}10^{-3}$	$20.4{\cdot}10^{-3}$
Damping [dB]	approx. 33.2	approx. 33.8

The applied algorithm allowed for more accurate determination of the harmonic component signal frequency. For the parameters applied, the accuracy of analysis of the obtained rotational frequency was ± 0.015 Hz (while the value obtained from the narrowband spectrum was ± 1.53 Hz). Differences in the coefficients of amplification values were above 6%. This more accurate analysis allowed to limit the influence of the lower-energy signal components which in the classical analysis are taken into account in the total energy represented by the spectral line.

5. Uncoupling equations of the vibroacoustic energy propagation

The proposed algorithm can be utilised in two ways. When we have a possibility of performing a certain number of partial measurements (including possibility to switch off temporarily some vibroacoustic energy sources), determination of unknowns from the system of Eqs. (9) is possible and thus also the relatively accurate model description (Fig. 5). When we do not have such possibility, more accurate determination of parameters of harmonic components allows to replace the relatively complicated non-linear model with the parallel linear model. In such case, maintaining constant excitation, i.e. stationary motion of the system, is indispensable (it results from the general theory of non-linear systems) (Batko *et al.*, 2008). Equations (9) are then reduced to the form

$$\begin{aligned}
\mathsf{S}\,[N_1]\cdot|H^*_{N1X}| &= \mathsf{S}\,[X]_{N1}\cdot\varPhi_{N1X},\\
\mathsf{S}\,[N_2]\cdot|H^*_{N2X}| &= \mathsf{S}\,[X]_{N2}\cdot\varPhi_{N2X},\\
\mathsf{S}\,[V_1]\cdot|H^*_{V1X}| &= \mathsf{S}\,[X]_{V1}\cdot\varPhi_{V1X},\\
\mathsf{S}\,[V_2]\cdot|H^*_{V2X}| &= \mathsf{S}\,[X]_{V2}\cdot\varPhi_{V2X},\\
\sum_i \mathsf{S}\,[X]_{Ni} + \sum_j \mathsf{S}\,[X]_{V1j} + \sum_i \varPhi_{NiX} & \\
+ \sum_j \varPhi_{VjX} + \varPsi &= X,
\end{aligned} \tag{14}$$

where S – operator denoting operations performed according to the proposed algorithm, $\mathsf{S}\,[N_i]$ – dominating signal harmonic components of the source N_i, $\mathsf{S}\,[N_j]$ – dominating signal harmonic components of the source V_j, $\mathsf{S}\,[X]_{Ni} = U_{NiX}$ – fraction of dominating harmonic components of the source N_i in the observed process measured at the system output within an accuracy of the value of function $\varPhi_{NiX}$, $\mathsf{S}\,[X]_{Vj} = U_{VjX}$ – fraction of dominating harmonic components of the source V_j in the observed process measured at the system output within an accuracy of the value of function $\varPhi_{VjX}$,

$$\begin{aligned}
|H^*_{N1X}| &= |H_{N1X} + H_{N1N2}\cdot H_{N2X}|,\\
|H^*_{N2X}| &= |H_{N2X} + H_{N2N1}\cdot H_{N1X}|,\\
|H^*_{V1X}| &= |H_{V1X} + H_{V1N1}\cdot H_{N1X}|,\\
|H^*_{V2X}| &= |H_{V2X} + H_{V2N2}\cdot H_{N2X}|,
\end{aligned}$$

and meaning of the remaining symbols is the same as in Eq. (7).

Equations (14) are uncoupled within an accuracy of the value of function $\varPhi_{ijX}$, and after transferring into the logarithmic scale (values in decibels), they can be reduced to equations representing balancing out the level decreases on each of the propagation paths and the equation determining the total level in the cabin:

$$\begin{aligned}
L_{N1} + L_{HN1X} &= L_{UN1} + \Delta_{N1X},\\
L_{N2} + L_{HN2X} &= L_{UN2} + \Delta_{N2X},\\
L_{V1} + L_{HV1X} &= L_{UV1} + \Delta_{V1X},\\
L_{V2} + L_{HV2X} &= L_{UV2} + \Delta_{V2X},\\
\log_{10}\left(\sum_i 10^{L_{UNiX}} + \sum_j 10^{L_{UVjX}}\right) + \Delta_X &= L_X,
\end{aligned} \tag{15}$$

where L_{Ni} – level of dominating harmonic components of the source N_i, L_{Vj} – level of dominating harmonic components of the source V_j, L_{HNiX} – decrease of the signal level between the source N_i and system output X, L_{HViX} – decrease of the signal level between the source V_j and system output X, L_{UNi} – fraction level of dominating harmonic components of the source N_i in the observed process measured at the system output within an accuracy of the value of ΔN_{iX}, L_{UVi} – fraction level of dominating harmonic components of the source V_j in the observed process measured at the system output within an accuracy of the value of ΔV_{jX}, L_X – signal level at the system output, Δ_{NiX} – relative error of separating the propagation path between the source N_i and the system output, Δ_{VjX} – relative error of separating the propagation path between the source V_j and the system output, Δ_X – total error of separating the propagation paths between the sources and the system output.

6. Conclusions

On the basis of the performed investigations it can be stated that the proposed algorithm utilising the normal coherence function allows to solve the identification problem of the general model of the noise and vibrations in non-linear multi-source system, at least for harmonic components. It is possible 'to assign' to a process, observed at the output, the components originated from individual sources with a high accuracy, even in the most difficult case of two twin-type driving systems operating with the same nominal rotational speed. Thus, it allows to calculate the coefficients of amplification on the source-system output path.

The application of the intermediate method (with a reference signal) allows to eliminate or reduce errors of the correlation function estimations which could be caused by:

- presence of external disturbances affecting input or output signals;
- non-linearity of the system binding signals at inputs and outputs;
- occurrence of feedbacks in the investigated system;
- interactions between the sources;
- correlation between input processes (presence of identical or similar harmonic components in various input processes).

The identified model provides a simple tool for assessing the impact of additional sound- and vibration-isolating elements on the noise in the cabin, and thus the selection of suitable insulating structures. Then one can use Eqs. (15) by adding level decreases resulting from characteristics of those structures to the levels L_{HNiX} and L_{HViX} (Dąbrowski, 1992).

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