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The influence of the piezoelements placement on the active vibration damping of smart truss

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Vibrations control in truss structures reaches a great practical interest, mainly in modern structures of huge space vehicles and aircrafts. Two demands essences are required in design of such structures. The first is the excellent dynamic behavior, in order to guarantee the stability of the structure and high precision pointing. The second is the necessity to obtain light structures, in order to reduce the cost. However, these two requirements are often contradictory, because light structures have low degrees of internal damping, which hinders the accuracy requirements.

These difficulties can be overcome by applying recently developed advanced materials, as for instance piezoelectric materials. Several researchers have proved that piezoelectric material can effectively counteract the vibrations. Appropriate assurance active damping of these vibrations can be achieved by optimal location of the piezo elements in the structure.

Key words: vibrations, piezo-stack actuator, active truss, modal mass, modal stiffness

1. Introduction

The vibrations control of the truss structures reached a great practical interest in many industrial applications, e.g. modern structures of the huge space vehicles, aircrafts, etc. Two demands are required in design of such structures. The first is the excellent dynamic behavior in order to guarantee the stability of the structure and high precision pointing. The second is the necessity to obtain light structures, which allow for reduction of the cost. However, these two requirements are often contradictory. The weak internal damping in light structures hinders the accuracy requirements [1].

These difficulties can be overcame by applying recently developed modern materials, for instance, piezoelectric materials. Several researchers have proved that piezoelectric materials can effectively counteract the vibrations. Of course, the appropriate active damping of these vibrations can be achieved by the optimal location of the piezoelements in the structure.

In this paper we consider the problem of location of the actuators that affects the values of the control forces. First, dynamics of the steel space truss without and with two

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sticked into a bar piezo-stack actuators is investigated. The truss dynamic is calculated for different locations of the actuators along the vertical axis of the structure with use of finite element solution. The natural frequencies and shapes of the first four modes, modal masses and modal stiffness are calculated. Next, the modal control forces for different locations of the piezo-stacks were found. Finally, the correlations between these variables for four lower vibration modes were calculated.

Through above described investigations we can obtained the quasi-optimal location of the piezo stacks. This allows for avoiding the problems connected with modeling: damping coefficients, glue connected piezo-elements with steal elements, modeling voltage amplifier and other automation elements.

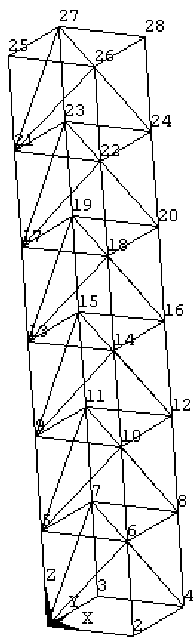

Figure 1. The experimental active truss.

2. Finite element modeling of the active truss

Consider the active truss with the piezoelectric stacks sticked into bars in one plane *Y-Z* and two sensors (accelerometers) glued on top of the truss (nodes 26 and 28) as it is shown in Fig. 1. The dynamics of the space truss without and with piezo-elements was calculated by using the finite element method (FEM). The parameters of the structure are presented in Table 1.

Table 7. Parameters of structure model

Parameters	**Steel bar** (vertical & horizontal bar)	**Steel bar** (crossbar)	**Piezo-actuator**
material	steel	steel	piezo-cermaic
dimensions (l) [m]	0.007× 0.007× 0.15	0.007× 0.007× 0.2121	0.006× 0.006× 0.010
cross section [m^2]	49e-6	49e-6	36e-6
density (ρ) [kg/m^3]	7800	7800	7900
Young's modulus (E) [GPa]	200	200	44
Poisson's coeff. (∋)	0.3	0.3	0.34

Table 8. First four natural frequencies of the truss

frequencies	**steel truss** [Hz]	**smart truss** [Hz]	**decrease** [%]
f_1	99.50	96.80	2.71
f_2	500.90	499.00	0.38
f_3	1099.60	1098.00	0.15
f_4	1665.60	1665.20	0.1

The truss consists of 28 nodes and 78 elements. The first four nodes are fixed and others are free. Dimensions of the truss are: 0.15 × 0.15 × 0.9 [m] (*length* × *width* × *high*). To design model we have used Ansys software with LINK 8 elements for the steel parts and for the piezo-stacks. A bonding between the truss and actuators are described by the elastic elements COMBIN 14. The first four modes of the examined truss were calculated. Obtained results are shown in Fig. 2. This figure represents the mode shapes of the steel truss without piezo-elements. Similar mode shapes was obtained for the same truss with piezo-stacks located at the above described points (smart truss).

The first 4 natural frequencies of the steel truss and the smart truss are given in Table 2. As expected, the reduced stiffness of the active bars, caused by piezo-stack actuators sticked into them, results in lower natural frequencies of the smart truss. The decrease of the natural frequencies are rather slight, as the height of piezo-stack is rather small when compared with the height of the whole truss (the ratio is 0.011).

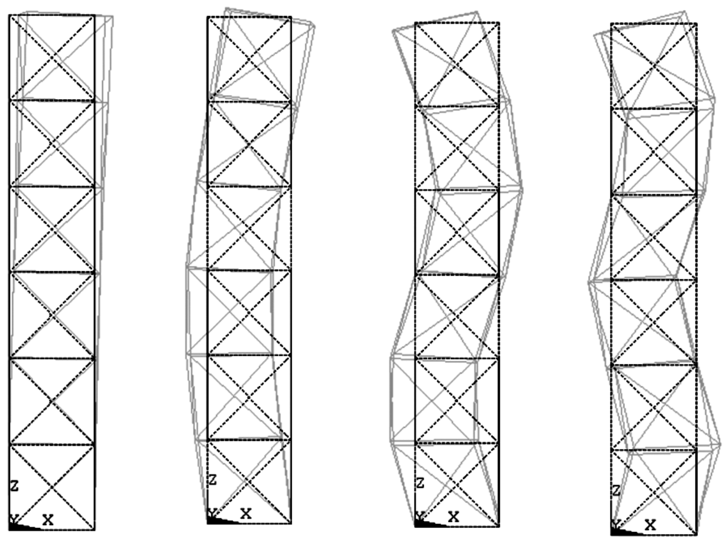

Figure 2. Mode shapes of the examined truss.

3. The influence of the piezoelement position on the natural frequencies of the truss

The optimal location of the piezo-stacks in the structure has the crucial influence on the vibrations damping. It is difficult problem in the real structures. Therefore for the case of piezo-elements with distributed parameters we are looking for a simple method which allows for finding their optimal location without calculations of the control forces. Thus, we have to find out how the location of the elements affects the natural frequencies of the smart truss. The piezo-stack actuators were moved from the fixed end to the opposite end with step equals to the length of the piezo-stack. Then, our calculations were repeated 78 times, to find the first four natural frequencies for these positions.

The changes of the natural frequencies are presented in Fig. 3. Only the first natural frequency monotonically raises when the actuators are moved from the fixed end of the beam. Other three frequencies vary around average values. It should be noticed that number of nodal points is twice greater in comparison with the number of nodal points of the mode shapes. Because the natural frequencies are function of the modal mass and the modal stiffness, in the next sections we should find these parameters also.

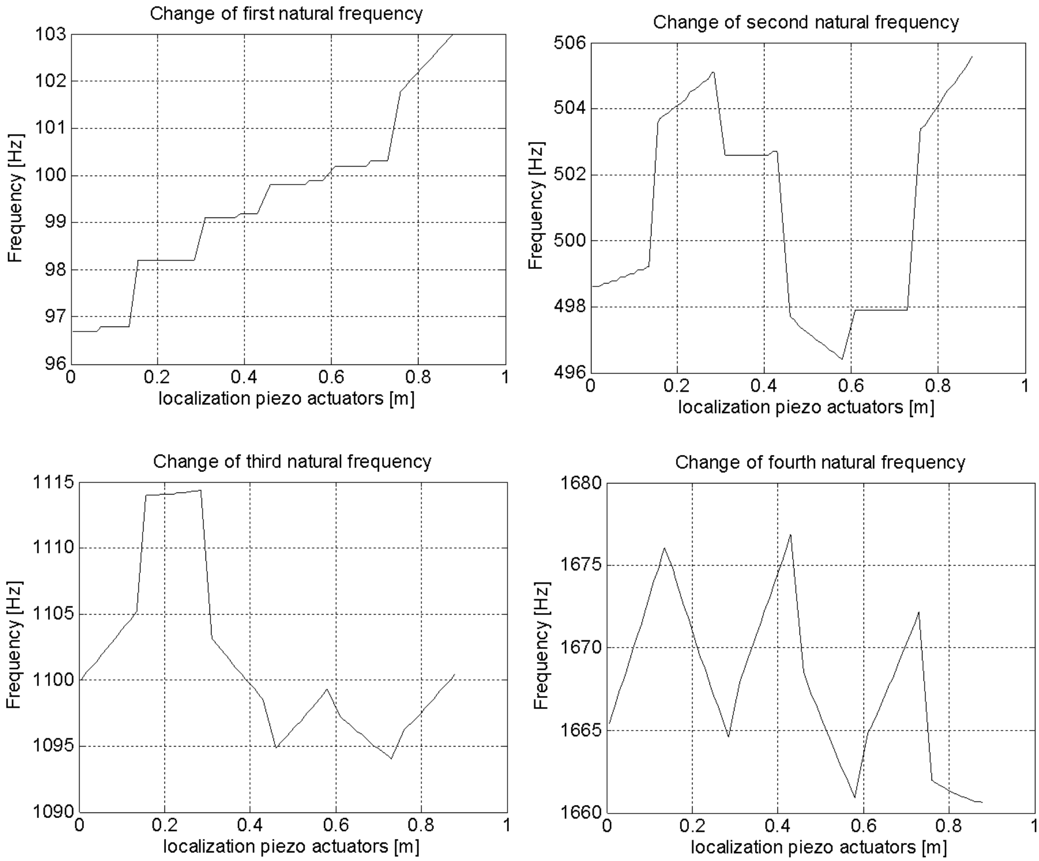

Figure 3. Natural frequencies of the smart truss versus the piezo-elements position.

4. Modal mass and stiffness

The main goal of this section is to obtain the modal masses and the modal stiffness as a function of the piezo-stacks location. Therefore, we have to derived the global mass matrix $M[r \times r]$ and the stiffness matrix $K[r \times r]$ to design the motion equation [2,3]:

$$M\ddot{x} + Kx = 0. \tag{1}$$

In order to design matrices M and K we have used free-body symmetric matrices M_{poz} and K_{poz} of the size $3n \times 3n$, (where n is the number of nodes) and boundary conditions matrix G [4]. Matrices M_{poz} and K_{poz} are constructed from the finite element parameters. Matrix G is derived with the use of boundary parameters of all finite elements. Finally, the global matrices can be represented in the form:

$$M = G^T M_{poz} G, \tag{2}$$

$$K = G^T K_{poz} G. \tag{3}$$

In the next step, the Matlab software was used to obtain the modal parameters. First, we have solved the eigenvalues problem to find the transformation (modal) matrix φ. Next, the modal masses m and the modal stiffness k parameters were calculated according to the following formula:

$$m = \varphi^T M \varphi \tag{4}$$

$$k = \varphi^T K \varphi \tag{5}$$

This procedure was repeated 78 times for different locations of the piezo-stack in the structure. In this way we obtained the modal mass and the modal stiffness parameters for the first 4 natural frequencies which are shown in Fig. 4 and Fig. 5.

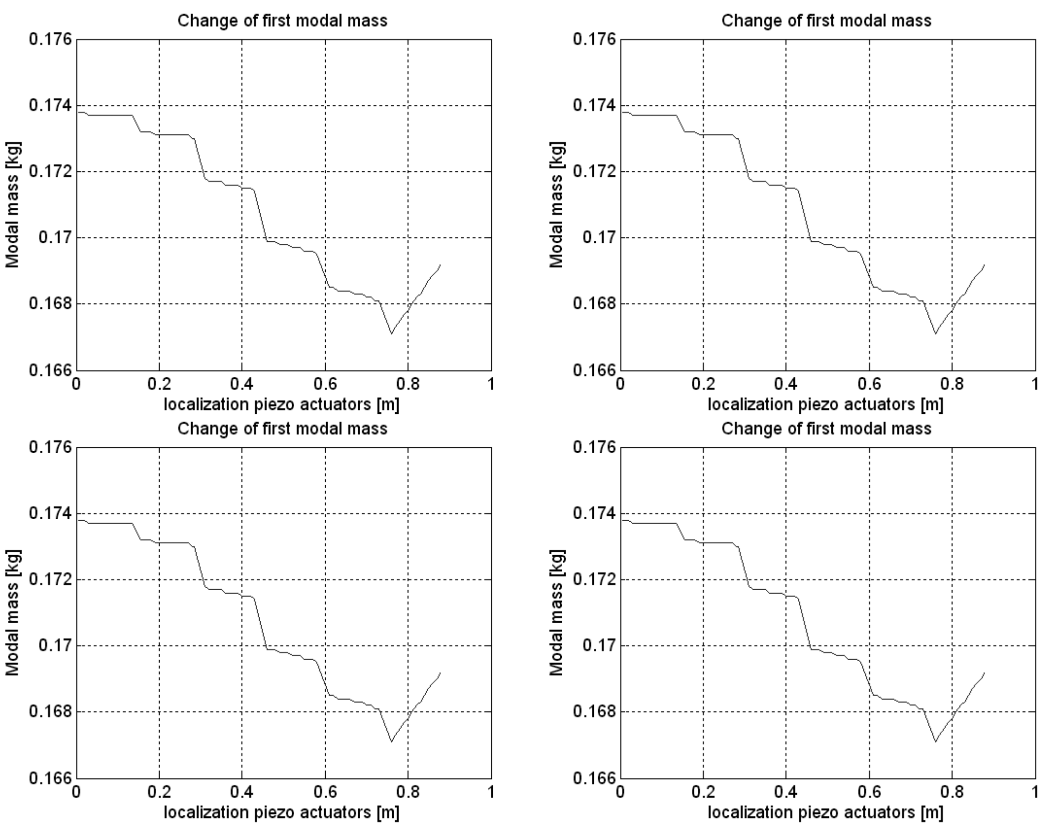

Figure 4. Modal mass versus the piezo-stack actuators localization.

Results obtained for the modal mass are very similar to the results for the natural frequencies but they have the opposite phase. So we can notice that changes of the frequencies strongly depend on changes of the modal mass.

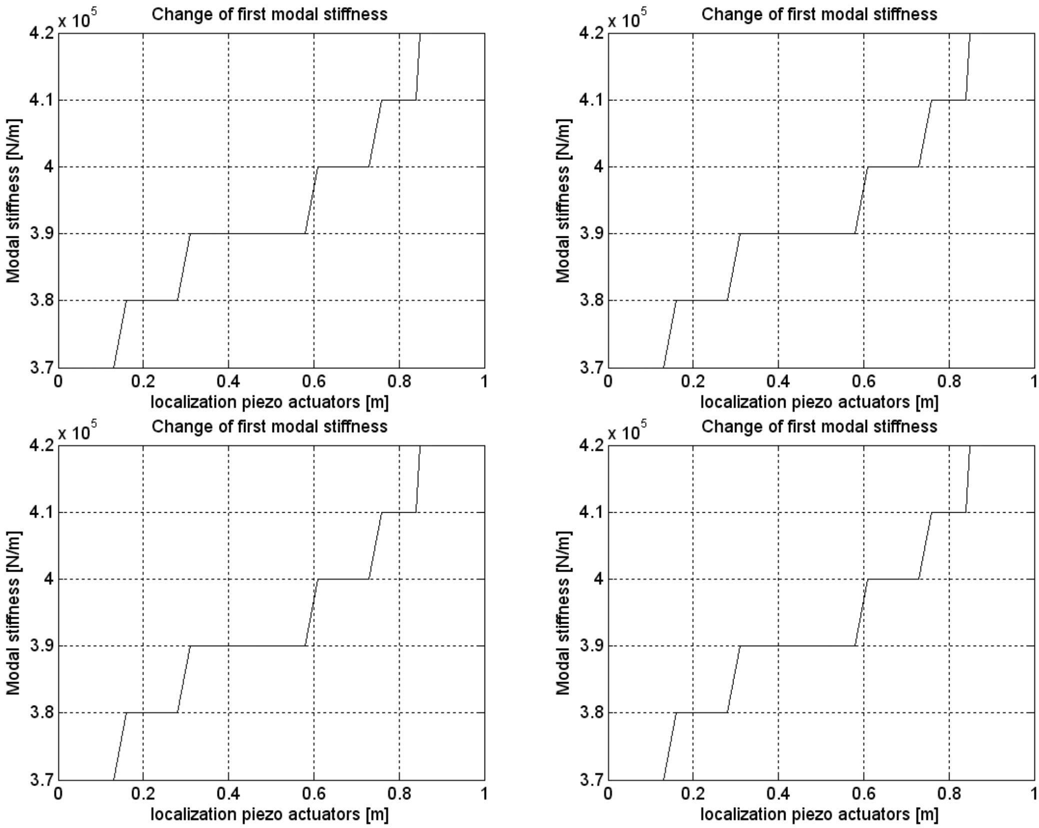

Figure 5. Modal stiffness versus the piezo-stack actuators localization.

5. Modal control forces

It is well known that location of the actuators have a great influence on the vibrations control system of the mechanical structures. Therefore, the question arises whether the obtained results provide information concerning the best position of the active member in the structure. This motivates the consideration of the smart truss where control forces proceed from piezo-stacks act on the opposite site in direction axis Z. The control forces action on the smart truss is presented in Fig. 6.

Taking into account the piezo-stacks dynamic, the motion equation has the form as follow [5,6,7]:

$$M\ddot{x} + Kx = F_{ctrl} \tag{6}$$

$$F_{ctrl} = B_0 \cdot V \cdot K_a \cdot d_{33} \cdot n \tag{7}$$

where:

B_0 – control coefficient matrix of the truss (non zero elements in cosine direction in global coordinates - see Fig.6),

K_a – stiffness of active member,

d_{33} – piezoelectric constant ($d_{33} = 450e - 12$ [m/V]),

n – amount of discs in piezo-stack ($n = 100$),

V – applied voltage vector to piezoelemant ($V = 100$ [V]).

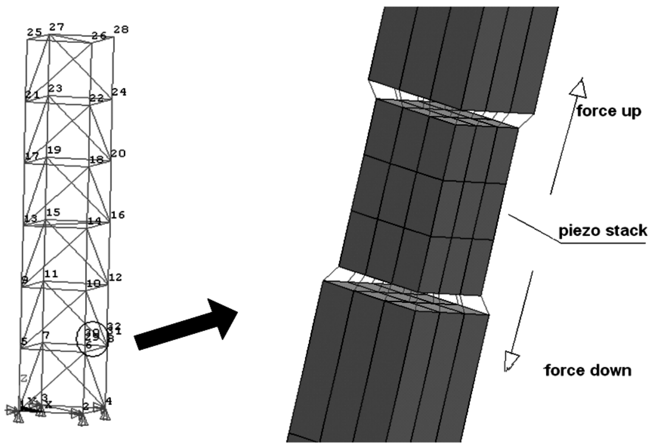

Figure 6. Active piezo-elements in the smart truss.

To obtain the control forces, the Matlab software was used. In the first step we calculated the control force for the above parameters taking into account the stiffness of the active member. Ten milimeters low voltage piezoelectric elements was applied. Elements consist of 100 piezo-discs with thickness 0.1 [mm] of single disk. According to equation (8) the four lower modal control forces for piezo stacks was calculated as follows:

$$f_{ctrl\,i} = \varphi_i^T F_{ctrl} \tag{8}$$

where i is the number of modal control force.

The main goal of the above calculations was to show the change of the control forces for four lowest natural frequencies to be further investigated. In this way we moved both piezo-stack actuators along vertical axis (Z) from the fixed end to free end with step equal 0.005[m]. It can be seen in Fig.7 that changes in the control forces (envelops obtained signals) have similar character then the mode shapes which were shown in section 2. It

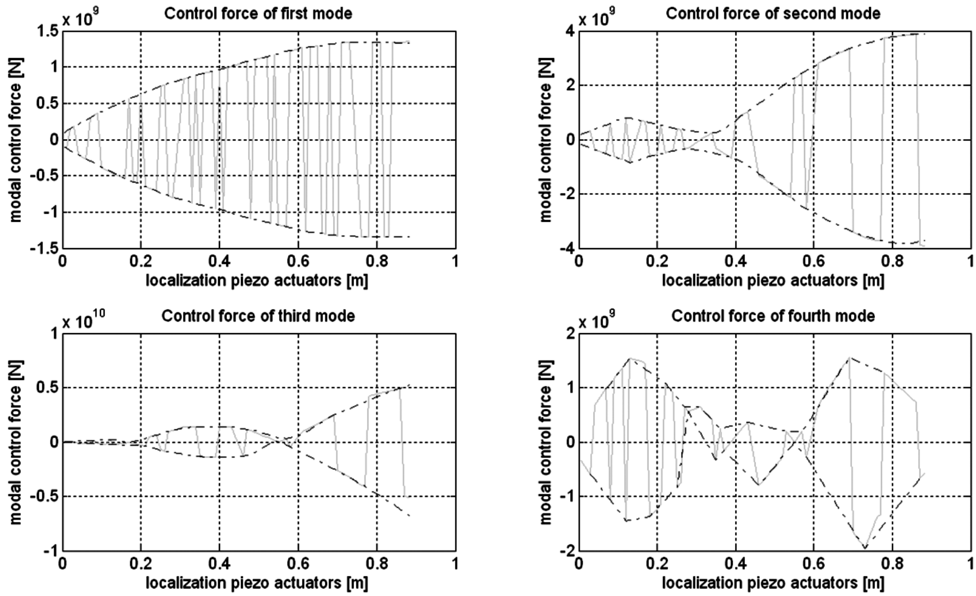

Figure 7. Forces generated by piezo-stack actuator.

follows from the requirement to control three lowest mode shape without fourth mode shape that we search in the base plots (Fig.7) such nodal points where the value of the control force is equal to zero. The fourth mode is not actuated to vibrate. Summarizing these calculations we can suppose that the best position of the both piezo-stack actuators for further control space truss will be in 0.32 [m] or 0.56 [m] from fixed end of the truss.

6. Correlation between obtained results

The results presented in the previous sections are now completed and compared in this chapter. A comparison tool will be the correlation matrices. The Matlab software was used to calculate four such matrices. The particular matrices are connected with the four lower vibration modes. Every matrix has seventy eight rows which represent the start position of the piezoelectric stack on vertical axis of the smart truss and four columns which contain values of: natural frequency changes, mass modal and stiffness modal respectively. Using these matrices we obtained the correlation coefficients between changes of natural frequencies, modal mass, and modal stiffness for each vibration mode. The correlation coefficients are presented in Tables 3-6.

We can notice that some variables are strongly correlated. Specially it can be seen in all correlations matrices between modal mass and natural frequency change and between modal stiffness and frequency change. The main reason to determine correlation matrices was to explain why the considered relation between frequency changes being caused

Table 9. Correlation coefficients for the first mode

	Freq. change	**Modal mass**	**Modal stiffness**
Freq. change	1.0000	**-0.8984**	**0.9840**
Modal mass	**-0.8984**	1.0000	**-0.8968**
Modal stiffness	**0.9840**	**-0.8968**	1.0000

Table 10. Correlation coefficients for the second mode

	Freq. change	**Modal mass**	**Modal stiffness**
Freq. change	1.0000	**-0.7841**	**0.99990**
Modal mass	**-0.7841**	1.0000	**-0.4865**
Modal stiffness	**0.9990**	**-0.4865**	1.0000

Table 11. Correlation coefficients for the third mode

	Freq. change	**Modal mass**	**Modal stiffness**
Freq. change	1.0000	**-0.7998**	**0.9419**
Modal mass	**-0.7998**	1.0000	**-0.7987**
Modal stiffness	**0.9419**	**-0.7987**	1.0000

Table 12. Correlation coefficients for the fourth mode

	Freq. change	**Modal mass**	**Modal stiffness**
Freq. change	1.0000	**-0.6466**	**0.9244**
Modal mass	**-0.6466**	1.0000	**-0.6090**
Modal stiffness	**0.9244**	**-0.6090**	1.0000

with position changes of the piezo elements contains twice more nodal points than the corresponding modes. These matrices allow for conclusion that their entries are strongly correlated and that a specially high correlation concerns frequency change and modal stiffness.

7. Conclusion

In this paper we have shown the dependence between natural frequencies, modal masses, modal stiffness and modal control forces for different locations of the piezo stack actuators along the truss. The calculations aided with the finite elements method (FEM) have indicated strong relations between modal mass and natural frequency changes. Fig.3 and Fig. 4 show that these variables have similar character of the changes. The values of natural frequencies decrease when the piezo stack actuator was placed in the truss member.

The results obtained in section 5 give information concerning the best localization of the piezo-elements on the structure. On the basis of Fig. 7 one should suppose, that in the case of three lowest mode control, the best location of the piezo-stacks is the nodal point of the fourth mode. If the piezo-element is fixed to the structure in this position then the design procedure of the active vibration damping system can be conducted faster and easier and experiments can be realized in the following three steps:

- Identification of the open loop model to obtain a model of the control plant without influence of the spillover effects in experimental investigations.
- Design the proper control law.
- Verification of the control system.

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